

# Evaluating Limits More Techniques

So far, **Direct Substitution** for **Well-Behaved Functions**

Constant

Linear

Polynomial

Sine/Cosine

Rational (if denominator  $\neq 0$ )

Radical (if radicand  $\geq 0$ )

# Not well behaved?

lim

$x \rightarrow c$

divide by zero  
 $\sqrt{\text{of a neg}}$

**Options:**

- 1) Factor
- 2) Rationalize (conjugate)
- 3) Simplify (common denominator)
- 4) Trig Identities
- 5) Expand

## Other Techniques

Divide out

$$\lim_{x \rightarrow 7} \frac{x-7}{x^2-49} = \left( \frac{1}{14} \right)$$

$$\frac{\cancel{x-7}}{(\cancel{x-7})(x+7)}$$
$$\frac{1}{x+7}$$

Try  $\lim_{x \rightarrow -2} \left( \frac{2x^2 + 3x - 2}{x+2} \right) = (-5)$

$$\frac{(\cancel{x+2})(2x-1)}{\cancel{x+2}}$$

Rationalize

$$\lim_{x \rightarrow 2} \left( \frac{4 - \sqrt{18-x}}{x-2} \right) = \left( \frac{1}{8} \right)$$

$$\frac{(4 - \sqrt{18-x})(4 + \sqrt{18-x})}{(x-2)(4 + \sqrt{18-x})}$$

$$\frac{16 - 18 + x}{(x-2)(4 + \sqrt{18-x})} \rightarrow \frac{x-2}{(x-2)(4 + \sqrt{18-x})}$$

$$\frac{(3+i)(3-i)}{2(3-i)}$$

$$\text{Try } \lim_{z \rightarrow 0} \left( \frac{\sqrt{7-z} - \sqrt{7}}{z} \right) = \left( \frac{-1}{2\sqrt{7}} \right)$$

$$\frac{(\sqrt{7-z} - \sqrt{7})(\sqrt{7-z} + \sqrt{7})}{z(\sqrt{7-z} + \sqrt{7})} \rightarrow \frac{7-z-7}{z(\sqrt{7-z} + \sqrt{7})}$$

$$\frac{-z}{z(\sqrt{7-z} + \sqrt{7})} \rightarrow \frac{-1}{\sqrt{7} + \sqrt{7}}$$

Simplify

$$\lim_{x \rightarrow 0} \left( \frac{\frac{8}{x+2} - 4}{x} \right) = -2$$

$$\frac{\frac{8}{x+2} - \frac{4(x+2)}{x+2}}{x} = \frac{\frac{8 - 4x - 8}{x+2}}{\frac{x}{1}} = \frac{\frac{-4x}{x+2} \cdot \frac{1}{x}}{\frac{-4}{x+2}}$$

## Trig Identities

$$\lim_{x \rightarrow 0} \left( \frac{1 - \cos^2 x}{\tan^2 x} \right) = \textcircled{1}$$

$$\frac{\sin^2 x}{\frac{\sin^2 x}{\cos^2 x}} \rightarrow \frac{\cancel{\sin^2 x}}{1} \cdot \frac{\cos^2 x}{\cancel{\sin^2 x}}$$



Expand:

$$\lim_{x \rightarrow 0} \frac{-3x}{(-2x+2)^2 - 4} = \left( \frac{3}{8} \right)$$
$$\frac{-3x}{4x^2 - 8x + 4} \rightarrow \frac{-3x}{4x(x-2)} = \frac{-3}{4(x-2)}$$

## Exit Questions

Ques. 1: Evaluate:  $\lim_{x \rightarrow 5} \left( \frac{1}{x-5} \right)$  DNE

VA  $\uparrow$

Ques. 2: Evaluate:  $\lim_{x \rightarrow 3} (3x + 3) = 12$

Ques. 3: Evaluate:  $\lim_{x \rightarrow 2} \left( \frac{\sqrt{x+5} - 4}{x-2} \right) \frac{(\sqrt{x+5} + 4)}{(\sqrt{x+5} + 4)} = \frac{x+5-16}{(x-2)(\sqrt{x+5} + 4)}$

Ques. 4: Evaluate  $\lim_{x \rightarrow 4} \left( \frac{\cancel{x-4}}{(\cancel{x-4})(x+4)} \right)$

$\frac{1}{x+4} = \frac{1}{8}$

DNE



## Attachments

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sin x over x.ggb

$1+x^{-1}$ .ggb

piecewise.ggb